Rescaling and Parallel Time Integration for systems of Order Differential Equations

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Abstract

We consider a system of time-dependent Ordinary Differential Equations (ODE's), where one seeks $Y : [0, \infty) \to R^k$:

$$\frac{dY}{dt} = F(Y),\ 0 < t \leq T \leq \infty,\ Y(0) = Y_0. \tag{1}$$

Numerical (time) integration procedures to obtain approximate solutions to (1) are inherently sequential. All ODE's solvers are based on algorithms that "advance" a numerical process from a time t_k to t_{k+1}, where the sequence $\{t_k\}$ is usually a "refined" uniform or adaptive subdivision of $[0, T]$. Time-Parallel methods become interesting when one seeks to reduce the excessively high computational time of (1), where the time of existence is of very large order $T \approx \infty$. Time Parallelization is based on iterative Predictor-Corrector Schemes. Its success depends on appropriate predictions of the solution at the beginning of each "time slice" of some coarse grid $\{[T_{n-1}, T_n) | n = 1, 2, ..., N \leq \infty\}$, whereas $[0, T] = \cup_{n=1}^{N} [T_{n-1}, T_n]$. Good predictions would automatically reduce the total number of iterations N_{it}, $(N_{it} \leq N)$. A key performance indicator as to the high performance of the method should verify the criterion: $N_{it} << N$, i.e. the number of iterations should be much less than the number of slices.

On the basis of a multi-scaling technique that rescales both the variables Y and t on every slice, we introduce a concept of similarity which leads to a "ratio property" in the case when

$$(F(Y))_i = \Sigma_j a_{ij} Y_i^{k_{ij}} Y_j^{l_ij}.$$

Such situations occur when solving diffusion reaction problems an also Lotka-Voltera predators-preys logistics models.

In this paper, these concepts are applied to a second order initial value model:

$$y^{''} - |y^{'}|^{q-1}\ y^{'}\ +\ |y|^{p-1}\ y = 0,\ \ t > 0,\ y(0) = y_{1,0},\ y^{'}(0) = y_{2,0}. \tag{2}$$

This ODE describes the motion of a membrane. Specifically, when

$$p \leq q \leq \frac{2p}{p+1} \text{ and } p < 1, \tag{3}$$

the existence of the solution is global with a non-oscillatory blow-up at ∞, i.e.:

$$\lim_{t\to\infty} |y(t)| = \lim_{t\to\infty} |y^{'}(t)| = \infty, \tag{4}$$

the roots of y and $y^{'}$ being both countably infinite sets.

We show through slices rescaling, that similarity can be obtained with a ratio property leading to an efficient parallel time solver for (2).