NDU Math Competition for High School students 2019

Notre Dame University, Lebanon

Department of Mathematics and Statistics

You have 12 Questions

March 18, 2019

Question 1

Solve for x the equation

$$\Big|3 - 2|x|\Big| = x + 1$$

You have 5 minutes

Question 2

Calculate the following limit

$$\lim_{x \to \infty} \sqrt{x^2 + x + 1} - x$$

You have 5 minutes

Question 3

The line $y = -3x + 20$ is tangent to the graph of $y = f(x)$ at the point where $x = 8$. Find

$$\lim_{x \to 8} \frac{f(x) + 4}{\sqrt[3]{x} - 2}$$

You have 7 minutes

Question 4

If $134^m = 8$ and $67^n = 8$, find $\frac{1}{m} - \frac{1}{n}$.

You have 8 minutes

Question 5

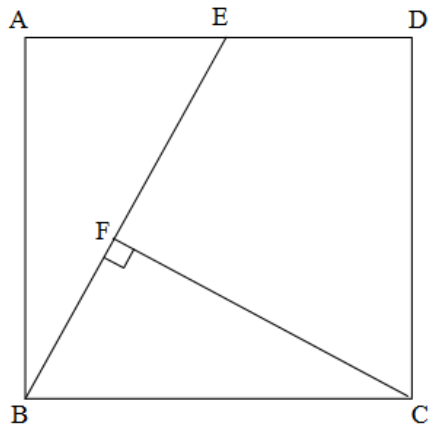
Let f be a decreasing function defined on $(0, +\infty)$.
Find all values of a in $\mathbb{R}$ that satisfy

$$f(2a^2 + a + 1) < f(3a^2 - 4a + 1).$$

You have 8 minutes

Question 6

ABCD is a a square such that $AB = 2$. E is the midpoint of $[AD]$. Find the area of $CDEF$.



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You have 8 minutes

Question 7

Simplify

$$-\frac{1}{2}\sqrt{1-\cos(4x)}\cos(x)+\cos\left(\frac{\pi}{4}\right)\left(\sin(x)\tan^2(x)\left(1+\cos(2x)\right)\right)$$

for $\frac{\pi}{4} < x < \frac{\pi}{2}$.

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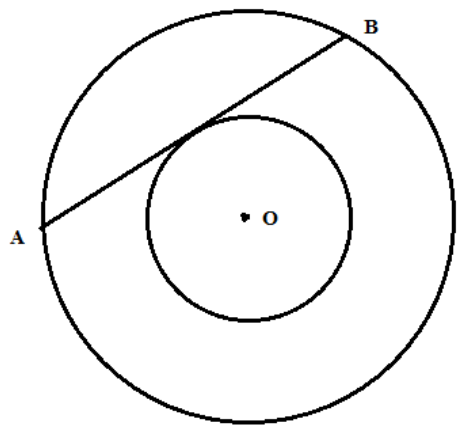
You have 6 minutes

Question 8

We have two concentric circles of radii 6 and 10 respectively. Find the length of the chord AB.



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You have 6 minutes

Question 9

Let $f : [a, b] \to \mathbb{R}$ be a differentiable function. Assume that there exists a $C > 0$ and some $\alpha > 0$, such that f satisfies

$$|f(x) - f(y)| \leq C|x - y|^{\alpha+1},$$

for all $x, y \in \mathbb{R}$. Prove that f is constant.

You have 8 minutes

Question 10

Show that

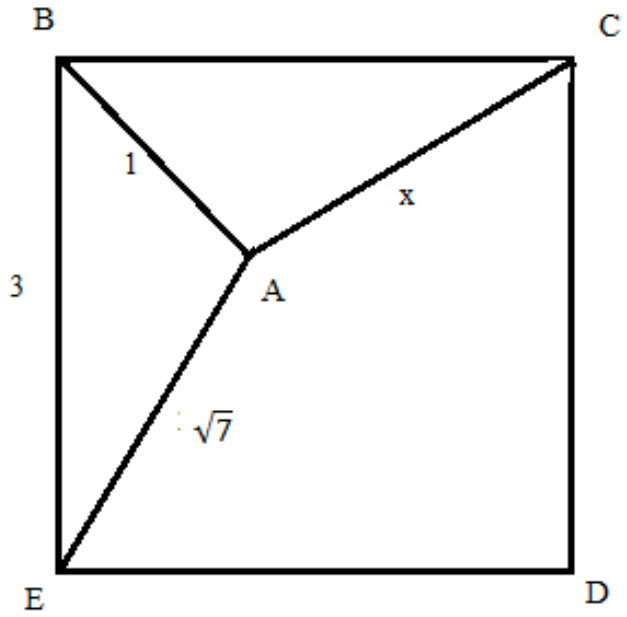
$$f(x) = x^3 + 2x + k$$

has exactly one real root, regardless of the value of $k \in \mathbb{R}$.

You have 8 minutes

Question 11

Let A be a point in the square $BCDE$. Calculate x.



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You have 8 minutes

Question 12

Let

$$\begin{cases} u_1 = 1 \\ u_{n+1} = 3u_n^2\,, \qquad \text{for } n \geq 1. \end{cases}$$

Find $\lim_{n\to\infty} u_n$ if it exists, or prove that (u_n) is divergent.

You have 10 minutes